the face of the planet are various distinct markings, amongst which it is easy to recognise the long "dumb-bell," drawn by Divini and Cassini, and figured in the first volume of the *Philosophical Transactions*. The satellite is only removed about $2\frac{1}{2}$ radii of *Mars* from the centre of the planet. The satellite is nearly four-tenths of the diameter of the primary, and both bodies are liberally provided with atmospheres.

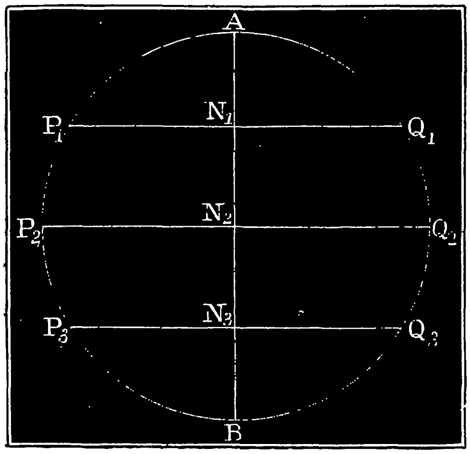
It may be mentioned that *Gulliver's Travels* were given to the world in 1726–7, while Voltaire's *Micromégas* seems to have been published about 1752. The statements they contain about the moons of *Mars* are widely known through Professor Hall's memoir. Kindermann's "discovery" is thus intermediate, in point of date, between the felicitous conceptions of the great satirists.

---

## *On the Illumination of Saturn's Rings during Sunrise; and on recent Observations of their Reappearance.* By the Rev. A. Freeman.

In order to ascertain the significance of the American and European observations made in 1891 October and November, near the time when the Sun's centre was in the plane of *Saturn's* Rings, I have investigated expressions for the intensity of light cast by the Sun upon the rings, at the four stages when, to an observer at *Saturn's* centre, one-fourth, one-half, three-quarters, and the whole of the Sun's diameter has risen above the plane of the rings. Adopting the method given by Sir J. Herschel in the article on *Light* (*Encyc. Metrop.* § 44 sqq.) or in Parkinson's *Optics*, § 42; if with vertex at the centre of the rings, we draw a cone whose generators touch the spherical surface of the Sun, and consider the boundaries of the section of a hemisphere radius unity, concentric with the rings, intercepted between the cone and the plane of the rings; and if we project this intercepted surface orthogonally upon the plane of the rings, we have an exact representation of the intensity of Sun-light received upon unit area at the centre of the ring-plane. Moreover, if $2\theta$ be the circular measure of the apparent diameter of the Sun as seen from *Saturn*, the intensity thus measured will be of the order $\theta^3$, and will differ in fact from the average intensity of the whole ring by a term of the order $\frac{a^2}{\rho^2}\theta^4$, in which $\frac{a}{\rho}$ is the circular measure of the radius of the ring as seen from the Sun, *i.e.*, the circular measure of about $20''\cdot 2$, and $\theta$ is the circular measure of about $102''\cdot 3$. This small term may evidently be neglected. The surface of the rings is not truly plane, but an increase of brightness of any element of the surface will be balanced by a corresponding decrease on an opposite element, and so the average illumination of the whole surface will not be affected by the deviation from a plane.

In the annexed figure the circle represents the section of the hemisphere by the cone when the Sun has wholly risen. $P_1Q_1$, $P_2Q_2$, $P_3Q_3$, are the sections of this circle by the ring's plane when $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of the Sun's diameter has risen.




1. If the radius of the hemisphere be unity, the radius of the circle will be $\sin\theta$ and its area $\pi \sin^2\theta$, projected at an inclination $\frac{\pi}{2}-\theta$ upon the ring's plane the intensity of illumination for the Sun just wholly risen is measured by $\pi \sin^3\theta$.

2. When the diameter $P_2Q_2$ is on the ring's plane, the projected area is the segment of a circle radius unity bounded by a chord equal to $2\sin\theta$, its value is

$$\theta-\sin\theta\cos\theta, \text{ or } \tfrac{2}{3}\theta^3 \text{ nearly.}$$

3. When $P_1Q_1$, which is equal to

$$2\sqrt{1-\cos^2\theta \,.\, \tan^2\frac{\theta}{2}}$$

is upon the ring-plane, *i.e.*, when the altitude of A

$$\text{is } \frac{\theta}{2},$$

and

$$N_2N_1=\cos\theta \,.\, \tan\frac{\theta}{2}$$

the projected area is the *difference* between a circular segment and an elliptic segment having the same chord $P_1Q_1$, its value is equal to

$$2\int_{\cos\theta\,.\,\sec\frac{\theta}{2}}^{1}\sqrt{1-x^2}\,.\,dx-2\sin\frac{\theta}{2}\int_{\cos\theta\,.\,\tan\frac{\theta}{2}}^{\sin\theta}\sqrt{\sin^2\theta-x^2}\,.\,dx$$

Evaluated by the method of limits when $\theta$ is small, the value is

$$\left(\frac{3\sqrt{3}}{8}-\frac{\pi}{6}\right)\theta^3$$

4. When $P_3Q_3$ ($=P_1Q_1$) is upon the ring-plane, *i.e.*, when the altitude of A is $\frac{\theta_3}{2}$, and $N_2N_3=N_2N_1$, the projected area is the *sum* of a circular segment and an elliptic segment (greater than half the ellipse) having the same chord $P_3Q_3$; its value may be derived from the last from the consideration that the *difference* between it and the last area is the ellipse formed by projecting the circle $\pi \sin^2\theta$ at an inclination of $\frac{\pi}{2}-\frac{\theta}{2}$, *i.e.*,

$$\pi \sin^2\theta \,.\, \sin\frac{\theta}{2}=\tfrac{1}{2}\pi\theta^3 \text{ nearly.}$$

Hence the area we require is

$$\left(\frac{\pi}{2}+\frac{3\sqrt{3}}{8}-\frac{\pi}{6}\right)\theta^3, \text{ or } \left(\frac{\pi}{3}+\frac{3\sqrt{3}}{8}\right)\theta^3.$$

The intensities of the illumination of the ring-plane by the Sun are in the four stages proportional to

$$\left(\frac{3\sqrt{3}}{8}-\frac{\pi}{6}\right),\ \tfrac{2}{3},\ \left(\frac{\pi}{3}+\frac{3\sqrt{3}}{8}\right),\ \pi.$$

that is to say, as

$$0{\cdot}12592 : 0{\cdot}66667 : 1{\cdot}69672 : 3{\cdot}14159,$$

or nearly, as

$$4 : 21 : 54 : 100.$$

Now the Sun's diameter at the mean distance from the Earth is $32'\ 3''{\cdot}64$ and on October $30^d\ 12^h$ G.M.T. (astro.), which I shall show to be the date given by Bessel's corrected formula for the hour when the Sun's centre was in the ring-plane, the logarithm of *Saturn's* heliocentric distance was 0·9744038. Hence, if $x$ be the Sun's semi-diameter in seconds of arc, as seen from *Saturn* at this date,

$$L_{10} \sin x = L_{10} \sin 16'\ 1''{\cdot}82 - 0{\cdot}9744038,$$

whence

$$2x = 3'\ 24''{\cdot}66 = 204''{\cdot}66 = \text{Sun's diameter.}$$

But in the ten days between October 26 and November 5 the change of the Sun's altitude relative to the ring-plane was $0^\circ{\cdot}154$ according to Mr. Marth's ephemeris, or $55''{\cdot}44$ daily; and consequently the time required for the Sun's diameter to cross the ring-plane is

$$\frac{204''{\cdot}66}{55''{\cdot}44}\,.\,24^h = 88^h{\cdot}19 \text{ nearly.}$$

Each of the four stages of sunrise follows its predecessor by 22 hours very nearly. The altitude of the Earth above the ring-plane differs very little from 2° 5′ during the 88 hours, its change is only to the extent of about 1 per cent. in 22 hours. Hence, the light received from the ring by an observer on the Earth will not differ appreciably from the proportions

$$4 : 21 : 54 : 100,$$

corresponding to the four stages of sunrise.

In addition to the observations reported by Mr. Barnard at Mount Hamilton, Mr. Comstock at Madison, Professor Oudemans at Utrecht, and by myself near Sittingbourne, all of which are published in the current volume of the *Monthly Notices*, I should wish to refer to the observation made by M. Léon Guiot at Soissons, France, October $30^d$ $18^h$ (*Astro. M.T.* of Paris), on which occasion he saw the ring represented by a fine line extending only one-eighth of the equatorial diameter of *Saturn* beyond the following limb (*L'Astronomie*, 1892 January). M. Guiot has informed me by letter that "on the following morning the ring was visible on both sides thin and long." This is a valuable confirmation of the momentary observation at Utrecht 40 minutes earlier. M. Guiot in his letter says that the instrument employed was a refractor of 10 centimetres aperture magnifying 150 times. It is probably the same as that which he used to observe the conjunction of ♃ and ♀ 1892 February 6, and which is stated more exactly to have an aperture of 95 m.m., or $3\frac{3}{4}$ inches (see *L'Astronomie*, 1892 March).

Recording the observations in order of G.M.T. (astro.) we have to nearest $5^m$.

October $29^d$ $17^h$ $10^m$, *Oudemans* ($10\frac{1}{4}$in. O.G.) Clear. *Saturn* decidedly without ring. Confirmed by myself with 6in. O.G. $45^m$ later.

October $30^d$ $1^h$ $5^m$, *Barnard* (36in. O.G.) The rings were easily and distinctly visible as slender threads of light.

October $30^d$ $17^h$ $50^m$, *Guiot* ($3\frac{3}{4}$in. O.G.) A fine bright line followed the East limb to a distance of only $\frac{1}{8}$ diameter of *Saturn*.

October $30^d$ $23^h$ $15^m$, *Comstock* ($15\frac{3}{4}$in. O.G.) Rings plainly visible as continuous lines, extending on each side $\frac{3}{5}$ diameter of ball.

October $31^d$ $17^h$ $10^m$, *Oudemans* ($10\frac{1}{4}$in. O.G.) Clouds. *Saturn* visible for a moment; at both sides the ring is visible as a thin bright line. Confirmed by Guiot with $3\frac{3}{4}$in. $40^m$ later.

November $1^d$ $17^h$ $30^m$, *Freeman* (6in. O.G.) Bright blue ansæ seen on both sides of *Saturn* to a distance about $\frac{5}{8}$ of his diameter.

There can be no doubt that the great telescope of Mount Hamilton would exhibit the rings in the first stage of illumina-

tion rather more easily than the telescope I used would do so in the fourth stage, the *areas* of the object glasses being as 36 : 1, and the quantities of light from the rings as 1 : 25. It will be noticed that the interval of time between Mr. Barnard's observation and my own is $64^h\ 25^m$, whilst the interval between the first and fourth stages is $66^h$. Again, the interval between Mr. Comstock's observation and my own is $42^h\ 15^m$, whilst the interval between the second and fourth stages is $44^h$; in this case, too, the areas of the object glasses are as 7 : 1, and the light received as 1 : 5 nearly.

I am inclined to think that Mr. Comstock's observation was made at a point of time near to the instant when the centre of the Sun was in the plane of the rings; and that the Sun was almost if not quite fully risen above the plane of the rings when I saw the ansæ. Further observations from Observatories in Europe and Australia will, it may be hoped, yet be published, and contribute to determine the place of the node of the ring upon the ecliptic. I am decidedly of opinion that the appearances seen by Mr. Comstock and Mr. Townley, October 20, 25, 26, 27, are either views of the illuminated edges of the rings and of their principal divisions, when the Sun and Earth were on opposite sides of the rings, according to Bond's explanation in *Annals of Harvard Observatory*, vol. ii., Part I., pp. 118–120; or else, if G. A. Hirn's hypothesis be adopted, viz., that the rings are composed of separate satellites, *each with an atmosphere of its own*, revolving in circular orbits in planes inclined at diverse angles within small limits to the equator of *Saturn*, then the appearances seen at Washburn Observatory (previous to the real reappearance of the ring-plane caused by the beginning of actual sunrise) may have arisen from the light of the Sun being diffused through the interstices of this infinite number of satellites *by means of their atmospheres*, thereby giving a feeble illumination to that side of the ring-plane which was presented to the Earth. It is strange that no other observer but myself has stated the colour of the rings when first seen. This deficiency may yet be made good. To assist in the future discussion of the problem, I will add that the formulæ of Bessel, corrected by insertion of the truer values of $\delta i$, $\delta n$, and $m$, as found by Professor Oudemans (*Monthly Notices*, vol. xlix., p. 57), are

$$n = 167^\circ\ 18'\ 41''\cdot9 + 46''\cdot473\,(t - 1833)$$
$$\iota = 28^\circ\ 10'\ 32''\cdot1 + 0''\cdot351\,(t - 1833),$$

and for 1891 October $30^d\ 12^h = 1891\cdot8282$

$$n = 168^\circ\ 4'\ 15''\ 85, \quad \iota = 28^\circ\ 10'\ 11''\cdot45.$$

the change in one day or 0·0027 of a Julian year is unimportant. The *Nautical Almanac* gives for *Saturn*,

October $30^d\ 0^h$, Helioc Long. 172° 3′ 43″·0, Lat. 2° 8′ 34″·9.
October $31^d\ 0^h$, Helioc Long. 172° 5′ 46″·2, Lat. 2° 8′ 37″·6.

and upon the *hypothesis* of the node of the ring-plane on the ecliptic passing through the Sun's centre at either of these dates, the value of *i* being taken as on October $30^d\ 12^h$, we shall find for the longitude of the node corresponding to

October $30^d\ 0^h$......$168^\circ\ 3'\ 18''\cdot1$.
October $31^d\ 0^h$......$168^\circ\ 5'\ 16''\cdot2$.

from which it will appear that the value given by the formula of Bessel as corrected corresponds to October $30^d\ 11^h\cdot75$.

In conclusion, I may say that the most probable reasons for my not having seen any trace of the rings, October $30^d\ 18^h$, are (1) that I observed too short a time before sunrise, (2) that definition that morning was not nearly so good as on the two days previous, fewer belts being seen on the planet. The observations of Mr. Barnard and Mr. Comstock seem to have been made $81^m$ and $76^m$ before sunrise to them.

*Murston Rectory, Sittingbourne:*
1892 *May* 7.

---

*Negatives of Jupiter, made with the Great Telescope of the Lick Observatory during* 1891. By Edward S. Holden and W. W. Campbell.

We have the honour to present to the Royal Astronomical Society, in the name of the Lick Observatory, a series of negatives of *Jupiter*, made with the Great Telescope during the year 1891, in continuation of the work of 1890. A list of the negatives is given below. The following memoranda should accompany them. The negatives are all taken on 8 × 10 plates—Seed, No. 26. The enlargements are directly made, in the telescope, by means of an ordinary camera objective of 2 inches aperture and 14 inches focus, furnished us by Alvan Clark, Jun., for the purpose. This lens is used so as to give an enlargement of a little over eight diameters—that is, the negatives are affected by the wind blowing on the instrument (and by other accidental disturbances) as if they were taken in the principal focus of a telescope 400 feet long.

The enlargement employed is too great, and the enlarging lens itself is not entirely satisfactory. We hope to procure a new lens before the next opposition, which shall give better images and less magnifying.

All the exposures have been made by the two observers working together, and all the plates have been developed by Mr. Campbell. Each plate is intended to be marked with the date, the observers' initials, the setting of the enlarging lens in its tube (5·06), the setting of the negative plate on the focusing screw (1·0), the Pacific standard times of the beginning and